

# A de Sitter tachyon thick braneworld and gravity localization

Gabriel Germán<sup>‡,1</sup>, Alfredo Herrera–Aguilar<sup>‡,†,2</sup>,  
Dagoberto Malagón–Morejón<sup>†,3</sup>, Refugio Rigel Mora–Luna<sup>†,4</sup>  
and Roldão da Rocha<sup>‡,5</sup>,

<sup>‡</sup>Instituto de Ciencias Físicas, Universidad Nacional Autónoma de México.  
Av. Universidad s/n, Col. Chamilpa, MEX–62210, Cuernavaca, Morelos, México.

<sup>†</sup>Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo.  
Edificio C–3, Ciudad Universitaria, C.P. 58040, Morelia, Michoacán, México.

<sup>‡</sup>Centro de Matemática, Computação e Cognição, Universidade Federal do ABC.  
09210-170, Santo André, Sao Paulo, Brazil.

## Abstract

Among the multiple 5D thick braneworld models that have been proposed in the last years in order to address several open problems in modern physics, there is one that involves a tachyonic bulk scalar field. In this framework we study a thick braneworld with a cosmological background induced on the brane. It turns out that the field equations derived from the model with a warped 5D geometry are highly non-linear equations that admit a non-trivial solution for the warp factor and the tachyon scalar field with a de Sitter 4D cosmological background. Moreover, the non-linear tachyonic scalar field, that generates the brane in complicity with warped gravity, has the form of a kink-like configuration. However, the non-linear field equations' restricting character does not allow one to easily find thick brane solutions with a decaying warp factor which leads to the localization of 4D gravity and other matter fields. We manage to obtain such a thick brane configuration in this tachyon-gravity setup. When analyzing the spectrum of gravity fluctuations in the transverse traceless sector we show that 4D gravity is localized due to the presence of a *single* zero mode bound state separated by a continuum of massive Kaluza–Klein (KK) modes by a mass gap. This is in contrast with previous results where there is a KK massive bound excitation that has no clear physical interpretation. The mass gap is determined by the scale of the metric parameter  $b$ . We finally compute the corrections to Newton's law in this model and show that they exponentially decay and coincide with corrections reported in previous results within similar braneworlds with induced 4D de Sitter metric.

---

<sup>1</sup>E-mail: gabriel@fis.unam.mx

<sup>2</sup>E-mail: aha@fis.unam.mx

<sup>3</sup>E-mail: malagon@ifm.umich.mx

<sup>4</sup>E-mail: rigel@ifm.umich.mx

<sup>5</sup>E-mail: roldao.rocha@ufabc.edu.br

# 1 Introduction

Within the framework of the braneworld models embedded in a spacetime with extra dimensions and after the success of the thin brane models, where singularities are present at the position of the branes, in solving the mass hierarchy and 4D gravity localization problems [1, 2], it has become a matter of interest to find smooth braneworld solutions (for an interesting review see [3] and references therein). In some models, such solutions are obtained by introducing one or several scalar fields in the bulk and the large variety of scalar fields that can be used to generate these models gives rise to different scenarios [4]–[12]. By following this direction and by using the freedom one has to choose a scalar field, several authors have chosen a tachyonic scalar field in the bulk [11]–[15] and addressed issues like the mass hierarchy problem, and localization of gravity and matter fields in such a model with thin and thick branes. In the original Randall–Sundrum model, a Standard Model or TeV brane is introduced at a certain fixed distance, say  $r_c$ , from the gravitational or Planck brane in order to achieve the desired warping, and hence, resolve the hierarchy problem in a completely 5D geometrical way. However, this resolution mechanism gives rise to a new fine-tuning on the probe brane position, and therefore, to the need of stabilizing this brane separation. The stabilization of this brane separation is achieved through the Goldberger–Wise mechanism by associating to it a radion scalar field that models the radius of the fifth dimension when one ignores the back-reaction of the brane itself [16]. However, in [13] it was proved that when one takes into account the full back-reacted system, the brane separation becomes unstable when modeled by a standard scalar field even with an arbitrary self-interaction potential.

In order to solve this problem these authors proposed a tachyonic scalar field action for modeling and stabilizing the brane separation. They also obtained the desired warping from the Planck scale to the TeV one, thus resolving the fine tuning problem of the Higgs mass in a stable braneworld scenario and generalizing in a relevant way the Randall–Sundrum model (see also [15]). This fact physically motivates the use of a tachyonic scalar field within the braneworld paradigm since the back-reaction of the radion field must be taken into account in a self-consistent system.

On the other side, the thick brane configuration constructed in [14] possesses an increasing warp factor, since most of the attempts to solve the highly non-linear field equations leads to imaginary tachyon field configurations. This fact translates into delocalization of gravity and other kinds of matter like scalar and vector fields, while giving rise to localization of fermions. Alternatively, the localization of fermionic fields on these thick branes was investigated in [17] by using the thick brane solution found in [14] with an auxiliary scalar field that couples to the fermionic mass term. In this paper we propose a new thick brane tachyonic solution with a decaying warp factor that enables localization of 4D gravity as well as other matter fields. By analyzing the dynamics of the metric perturbations we see that their spectrum contains a *single* bound state corresponding to the 4D massless graviton of the model and a continuum of KK excitations separated by a mass gap. We further compute the corresponding correction to Newton’s law by analyzing the influence

of these massive modes on the gravitational potential acting between two point massive particles located along the center of the thick brane.

## 2 The thick brane model and its solution

The complete action for the tachyonic braneworld model is expressed as follows

$$S = \int d^5x \sqrt{-g} \left( \frac{1}{2\kappa_5^2} R - \Lambda_5 \right) + \int d^5x \sqrt{-g} V(T) \sqrt{1 + g^{AB} \partial_A T \partial_B T} \quad (1)$$

where the first term describes 5D gravity with a bulk cosmological constant  $\Lambda_5$ , the second is the action of the matter in the bulk,  $\kappa_5$  is the five-dimensional gravitational coupling constant, and  $A, B = 0, 1, 2, 3, 5$ . Thus, the tachyon field  $T$  represents the matter in the 5D bulk and depends, for simplicity, only on the extra dimension, while  $V(T)$  denotes its self-interaction potential [18].

The Einstein equations with a cosmological constant in five dimensions are given by

$$G_{AB} = -\Lambda_5 g_{AB} + \kappa_5^2 T_{AB}^{bulk}. \quad (2)$$

For the background metric we use the ansatz of a warped 5D line element with an induced 3-brane with spatially flat cosmological background that reads

$$ds^2 = e^{2f(\sigma)} \left[ -dt^2 + a^2(t) (dx^2 + dy^2 + dz^2) \right] + d\sigma^2 \quad (3)$$

where  $f(\sigma)$  is the warp factor and  $a(t)$  is the scale factor of the brane.

The matter field equation is obtained by variation of the action with respect to the tachyon and is expressed in the following form

$$T'' + 4f'T'(1 + T'^2) = (1 + T'^2) \frac{\partial_T V(T)}{V(T)}. \quad (4)$$

By using the ansatz (3) we can obtain the components of the Einstein tensor

$$\begin{aligned} G_{00} &= 3 \frac{\dot{a}^2}{a^2} - 3 e^{2f} (f'' + 2f'^2), \\ G_{\alpha\alpha} &= -2 \ddot{a}a - \dot{a}^2 + 3 a^2 e^{2f} (f'' + 2f'^2), \\ G_{\sigma\sigma} &= -3 e^{-2f} \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) + 6f'^2, \end{aligned} \quad (5)$$

where “ $\prime$ ” and “ $\dot{\phantom{x}}$ ” are the derivative with respect to the extra dimension and time, respectively, while  $\alpha$  labels the spatial dimensions  $x, y$  and  $z$ . The components of the stress energy tensor do not depend explicitly on time and read

$$T_{AB}^{bulk} = \left[ g_{AB} V(T) \sqrt{1 + (\nabla T)^2} - \frac{V(T)}{\sqrt{1 + (\nabla T)^2}} \partial_A T \partial_B T \right], \quad (6)$$

while the Einstein equations (2) can be rewritten in a simple way:

$$f'' = \kappa_5^2 \frac{V(T)T'^2}{3\sqrt{1+T'^2}} - e^{-2f} \frac{\ddot{a}}{a}, \quad (7)$$

$$f'^2 = \kappa_5^2 \frac{V(T)}{6\sqrt{1+T'^2}} - \frac{\Lambda_5}{6} + \frac{e^{-2f}}{2} \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right). \quad (8)$$

Thus, we end up with the case of a single brane configuration in a 4D spatially flat cosmological background given by equations (4), (7) and (8).

Now we take the restriction equation (8) and derive it with respect to the extra dimension to get

$$f'' = \frac{\kappa_5^2}{12} \frac{T' \partial_T V(T)}{f' \sqrt{1+T'^2}} - \frac{\kappa_5^2}{12} \frac{T' T'' V(T)}{f' (1+T'^2)^{\frac{3}{2}}} - \frac{e^{-2f}}{2} \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) \quad (9)$$

and then equate relations (7) and (9) in order to obtain the following expression

$$T'' + 4f' T' (1+T'^2) + \frac{6f' (1+T'^2)^{\frac{3}{2}} \left( \frac{\dot{a}^2}{a^2} - \frac{\ddot{a}}{a} \right)}{\kappa_5^2 V(T) T'} e^{-2f} = (1+T'^2) \frac{\partial_T V(T)}{V(T)}, \quad (10)$$

which must coincide by mathematical consistency with equation (4). This fact yields an expression of de Sitter type for the scale factor since (4) and (10) coincide if and only if  $a(t) = ke^{bt}$ , where  $b$  and  $k$  are integration constants. Thus, we must have a de Sitter 4D cosmological background defined by

$$a(t) = e^{bt}, \quad (11)$$

since the constant  $k$  can be absorbed into a coordinate redefinition. The last asseveration makes clear the role of the action for the tachyonic scalar field as a non-trivial 5D configuration which leads to a braneworld in which the induced metric on the brane is described by  $dS_4$  geometry.

Here it is convenient to go to conformal coordinates through  $dw = e^{-f(y)} d\sigma$  which leads to the following metric

$$ds^2 = e^{2f(w)} \left[ -dt^2 + a^2(t) (dx^2 + dy^2 + dz^2) + dw^2 \right]. \quad (12)$$

According to the authors of [14], after obtaining equations (4), (7) and (8), it is easy to obtain separated equations for the scalar field  $T$  and for the potential  $V(T)$  by the following procedure. It turns out that despite the high non-linearity of these field equations, the derivative of the tachyonic scalar field as well as the arbitrary potential  $V(T)$  can be expressed in terms of the warp and scale factors of the metric (and their respective derivatives) after some simple manipulations:

$$T' = \pm e^f \sqrt{\frac{f'' - f'^2 + b^2}{2 \left( f'^2 + \frac{\Lambda_5}{6} e^{2f} - b^2 \right)}}, \quad (13)$$

$$V(T) = \frac{3}{\kappa_5^2} e^{-2f} \sqrt{\frac{2(f'' + f'^2 + \frac{\Lambda_5}{3} e^{2f} - b^2)}{(f'^2 + \frac{\Lambda_5}{6} e^{2f} - b^2)}} \left(f'^2 + \frac{\Lambda_5}{6} e^{2f} - b^2\right), \quad (14)$$

where now “ $f$ ” stands for derivatives with respect to  $w$  and we have taken into account the form of the scale factor corresponding to (11). Thus, by determining a desired behaviour of the geometry we completely fix the dynamics of the tachyon field and vice-versa. However, the resulting solution must be real and have physical sense. This restriction turns out to be very demanding, since several warp factors with “convenient” behaviour lead to a complex tachyonic field  $T$  and/or self-interaction potential  $V(T)$ .

It is straightforward to see that the following warp factor

$$f(w) = -\frac{1}{2} \ln \left[ \frac{\cosh [b(2w + c)]}{s} \right], \quad (15)$$

where  $b$ ,  $c$  and  $s > 0$  are constants, is a solution of the Einstein and field equations if the tachyon scalar field adopts the form

$$T(w) = \pm \sqrt{\frac{-3}{2\Lambda_5}} \operatorname{arctanh} \left[ \frac{\sinh \left[ \frac{b(2w+c)}{2} \right]}{\sqrt{\cosh [b(2w + c)]}} \right], \quad (16)$$

while the tachyon potential is given by the following expression

$$\begin{aligned} V(T) &= \frac{\Lambda_5}{\kappa_5^2} \operatorname{sech} \left( \sqrt{-\frac{2}{3}\Lambda_5} T \right) \sqrt{6 \operatorname{sech}^2 \left( \sqrt{-\frac{2}{3}\Lambda_5} T \right) - 1} \\ &= \frac{\Lambda_5}{\sqrt{2}\kappa_5^2} \sqrt{1 + \operatorname{sech} [b(2w + c)]} \sqrt{2 + 3 \operatorname{sech} [b(2w + c)]}. \end{aligned} \quad (17)$$

In the last two equations we have set  $s = -6b^2/\Lambda_5$  with a negative bulk cosmological constant  $\Lambda_5 < 0$ , a fact that tells us that we have a  $dS_4$  brane embedded into an  $AdS_5$  [19] spacetime.

Thus, the set of equations (11), (15), (16) and (17) are the necessary and sufficient ingredients that give rise to a tachyonic thick brane solution. The warp factor has a decaying and vanishing asymptotic behaviour, whereas the tachyon scalar is real and possesses a kink-like profile. In contrast with solutions found in [14], here we manage to express the tachyon potential  $V(T)$  in terms of the tachyonic scalar field  $T$ . This potential has a minimum at the position of the brane, but it is negative definite as it can be seen from (17). In fact, since the tachyon field is bounded, the potential does not extend along the whole fifth dimension and remains real as well as bounded.

### 3 Gravity localization and corrections to Newton’s law

In order to study the metric and field fluctuations of our system we must perturb both of these entities. Since the relevant geometry of the four-dimensional background is de Sitter,

the fluctuations of the metric may be classified into tensorial, vectorial and scalar modes with respect to the transformations associated to the symmetry group  $dS_4$  [20]. This fact turns out to be very important since at first (linear) order these modes evolve independently and, hence, their dynamical equations decouple from each other, even when the perturbed Einstein and tachyon field equations were highly coupled non-linear equations. Making use of this fact, we shall study the dynamics of the tensorial metric fluctuations which have the physical interpretation of the five-dimensional braneworld graviton. These tensorial metric fluctuations are gauge invariant and will enable us to determine whether localization of four-dimensional gravity, and hence the physics of our four-dimensional world, is feasible or not within this model.

To begin with, we shall write the Einstein equations in the form:

$$R_{AB} = \frac{2}{3}\Lambda_5 g_{AB} + \kappa_5^2 \hat{T}_{AB}. \quad (18)$$

where we have made use of the reduced energy-momentum tensor

$$\hat{T}_{AB} = -\frac{2}{3}V\sqrt{1+(\nabla T)^2}g_{AB} + \frac{V}{\sqrt{1+(\nabla T)^2}}\left[\frac{g_{AB}}{3}(\nabla T)^2 - \nabla_A T \nabla_B T\right]. \quad (19)$$

In the language of the conformal metric coordinates (12) the 00-component of (18) reads

$$f'' + 3f'^2 = 3H^2 - \frac{2}{3}\Lambda_5 e^{2f} + \frac{\kappa_5^2}{3}e^{2f}\frac{2+(\nabla T)^2}{\sqrt{1+(\nabla T)^2}}V(T), \quad (20)$$

since

$$R_{00} = -3H^2 + f'' + 3f'^2 \quad (21)$$

and

$$\hat{T}_{00} = \frac{V}{3}e^{2f}\frac{2+(\nabla T)^2}{\sqrt{1+(\nabla T)^2}}. \quad (22)$$

Let us perturb the metric as follows

$$ds_p^2 = (g_{AB} + h_{AB})dx^A dx^B = (g_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu + e^{2f(w)}dw^2, \quad (23)$$

where  $g_{AB} = (g_{\mu\nu}, e^{2f(w)}) = e^{2f(w)}\bar{g}_{AB} = e^{2f(w)}(\bar{g}_{\mu\nu}, 1)$ ,  $\bar{g}_{\mu\nu} = \text{diag}[(-1, \delta_{ij}a^2)]$ ,  $\mu, \nu = 0, \dots, 3$  and  $i, j = 1, 2, 3$ . In this approach we shall impose the transverse traceless condition on these metric fluctuations:  $\nabla^\mu h_{\mu\nu} = h_\mu^\mu = 0$ , where  $\nabla$  is the covariant derivative operator with respect to the metric  $g_{AB}$ .

After some algebraic work, the linearized Einstein equations for the transverse traceless fluctuation modes adopt the following form

$$\delta R_{AB} = \frac{2}{3}\Lambda_5 h_{AB} - \frac{\kappa_5^2 V}{3}\frac{2+(\nabla T)^2}{\sqrt{1+(\nabla T)^2}}h_{AB}. \quad (24)$$

At this stage, it is extremely useful to define a new fluctuation variable  $h_{\mu\nu} = e^{2f}\bar{h}_{\mu\nu}$ . By making use of the conformal transformation for the Ricci tensor, in terms of the barred fluctuations we have

$$\delta R_{\mu\nu} = \delta\bar{R}_{\mu\nu} - \frac{3}{2}f'\bar{h}'_{\mu\nu} - \bar{h}_{\mu\nu}(f'' + 3f'^2) \quad (25)$$

where all the barred quantities are computed with respect to the metric  $\bar{g}_{AB}$ . On the other side, this quantity can also be computed as follows (see, for instance, [21])

$$\delta\bar{R}_{\mu\nu} = \frac{1}{2} \left( -\square\bar{h}_{\mu\nu} + \bar{\nabla}_A\bar{\nabla}_\mu\bar{h}_\nu^A + \bar{\nabla}_A\bar{\nabla}_\nu\bar{h}_\mu^A - \bar{\nabla}_\mu\bar{\nabla}_\nu\bar{h}_A^A \right), \quad (26)$$

where  $\square\bar{h}_{\mu\nu} = \bar{g}^{\alpha\beta}\bar{\nabla}_\alpha\bar{\nabla}_\beta\bar{h}_{\mu\nu} + \bar{g}^{55}\bar{\nabla}_5\bar{\nabla}_5\bar{h}_{\mu\nu}$ .

In order to compute the involved quantities in this expression we recall that the non-null Christoffel symbols are

$$\bar{\Gamma}_{ij}^0 = a^2 H \delta_{ij}, \quad \bar{\Gamma}_{j0}^i = H \delta_j^i.$$

Since  $\bar{\Gamma}_{AB}^5 = \bar{\Gamma}_{5A}^B = 0$  and  $\bar{h}_{5A} = 0$ , we have

$$\bar{\nabla}_5\bar{\nabla}_5\bar{h}_{\mu\nu} = h''_{\mu\nu}, \quad \bar{g}^{\alpha\beta}\bar{\nabla}_\alpha\bar{\nabla}_\beta\bar{h}_{\mu\nu} = \square_4\bar{h}_{\mu\nu}, \quad (27)$$

where  $\square_4$  is the d'Alembert operator on  $dS_4$ . While it is evident that  $h = 0$  implies  $\bar{h} = 0$  from their conformal relationship, it is not easy to see that  $\bar{\nabla}^\mu\bar{h}_{\mu\nu} = 0$  follows from  $\nabla^\mu h_{\mu\nu} = 0$  and we shall show here that this is indeed the case. On one hand, by definition of covariant derivative we have

$$\nabla^\mu h_{\mu\nu} = \bar{g}^{\mu\alpha}\nabla_\alpha\bar{h}_{\mu\nu} = \bar{g}^{\mu\alpha}(\partial_\alpha\bar{h}_{\mu\nu} - \Gamma_{\alpha\mu}^E\bar{h}_{E\nu} - \Gamma_{\alpha\nu}^E\bar{h}_{E\mu}).$$

On the other hand, from the relationship between the Christoffel symbols of two conformal metrics we get for our case

$$\Gamma_{BC}^A = \bar{\Gamma}_{BC}^A + (\delta_B^A\bar{\nabla}_C f + \delta_C^A\bar{\nabla}_B f - \bar{g}_{BC}\bar{\nabla}^A f).$$

From both of these relations we conclude that  $\Gamma_{\beta\gamma}^\alpha = \bar{\Gamma}_{\beta\gamma}^\alpha$  and, therefore, that  $\nabla^\mu h_{\mu\nu} = \bar{\nabla}^\mu\bar{h}_{\mu\nu} = 0$ .

Since in a curved spacetime the double covariant derivatives do not commute, for the metric fluctuations we have (see [22], for instance)

$$\bar{\nabla}_C\bar{\nabla}_\mu\bar{h}_\nu^C = \bar{\nabla}_\mu\bar{\nabla}_C\bar{h}_\nu^C + \bar{R}_{E\mu}\bar{h}_\nu^E - \bar{R}_{\nu C\mu}^E\bar{h}_E^C. \quad (28)$$

By making use of the transverse conditions  $\bar{\nabla}^\mu\bar{h}_{\mu\nu} = 0$  and the fact that  $\bar{\Gamma}_{AB}^5 = \bar{\Gamma}_{5A}^B = 0$ , we have

$$\bar{\nabla}_C\bar{h}_\nu^C = \bar{\nabla}_5\bar{h}_\nu^5 + \bar{\nabla}_\alpha\bar{h}_\nu^\alpha = 0, \quad \bar{\nabla}_\mu\bar{\nabla}_C\bar{h}_\nu^C = 0. \quad (29)$$

Since  $h_{5A} = 0$  and  $\bar{R}_{5ABC} = \bar{R}_{5A} = 0$ , we get

$$\bar{\nabla}_C \bar{\nabla}_\mu \bar{h}_\nu^C = \bar{R}_{\alpha\mu} \bar{h}_\nu^\alpha - \bar{R}_{\nu\beta\mu} \bar{h}_\alpha^\beta. \quad (30)$$

Moreover, for a  $dS_4$  spacetime the following relations hold

$$\bar{R}_{\mu\nu\alpha\beta} = H^2(\bar{g}_{\mu\alpha}\bar{g}_{\nu\beta} - \bar{g}_{\mu\beta}\bar{g}_{\nu\alpha}), \quad \bar{R}_{\mu\nu} = 3H^2 g_{\mu\nu}. \quad (31)$$

Hence, we get the following result for the linear variation of the Ricci tensor

$$\delta\bar{R}_{\mu\nu} = \frac{1}{2} \left( -\square_4 \bar{h}_{\mu\nu} - \bar{h}_{\mu\nu}'' + 8H^2 \bar{h}_{\mu\nu} \right) \quad (32)$$

Finally, by making use of the relations (20), (24), (25) and the last expression we have

$$\square_4 \bar{h}_{\mu\nu} + \bar{h}_{\mu\nu}'' + 3f' \bar{h}_{\mu\nu}' - 2H^2 \bar{h}_{\mu\nu} = 0, \quad (33)$$

under the imposed transverse and traceless conditions  $\bar{\nabla}^\mu \bar{h}_{\mu\nu} = 0$ ,  $\bar{h}_\alpha^\alpha = 0$ .

We further perform the following separation of variables for the metric fluctuations  $\bar{h}_{\mu\nu} = e^{-\frac{3}{2}f(w)} \Psi(w) \phi_{\mu\nu}(x)$  that brings the equation (33) into a Schrödinger-like one along the extra dimension:

$$\left( -\partial_z^2 + V_{QM} - m^2 \right) \Psi(w) = 0, \quad (34)$$

where the analog quantum mechanical potential  $V_{QM}$  reads

$$V_{QM} = \frac{9}{4} f'^2 + \frac{3}{2} f''. \quad (35)$$

On the other side, the 4D equation that we get from (33) is

$$\left( -\partial_t^2 - 3H\partial_t + e^{-2Ht} \nabla^2 - 2H^2 \right) \phi(x) = -m^2 \phi(x), \quad (36)$$

where  $m^2$  represents the mass that a 4D observer sees in a de Sitter spacetime [23]–[24] and we have omitted the indices of the function  $\phi_{\mu\nu}(w)$  for convenience.

By substituting the expression for the warp factor (15) into (35) the analog quantum mechanical potential adopts the form of a modified Pöschl–Teller potential that reads

$$V_{QM} = \frac{3b^2}{4} \left[ 3 - 7 \operatorname{sech}^2(2bw) \right]. \quad (37)$$

The fact that this potential possesses a definite positive asymptotic value ensures the existence of a mass gap in the graviton spectrum of KK massive fluctuations determined by  $\frac{9}{4}b^2$ , or equivalently  $m = \frac{3b}{2}$  [24]–[29].

By performing the following rescaling of the fifth coordinate  $v = 2bw$  we recast (34) into

$$\left( -\partial_v^2 - \frac{21}{16} \operatorname{sech}^2 v \right) \Psi(v) = \left( \frac{m^2}{4b^2} - \frac{9}{16} \right) \Psi(v), \quad (38)$$



which can be directly compared to the canonical form of the classical eigenvalue problem for the Schrödinger equation with a modified Pöschl–Teller potential

$$\left[-\partial_v^2 - n(n+1)\text{sech}^2 v\right] \Psi(v) = E \Psi(v) \quad (39)$$

with  $n = 3/4$  and  $E = \frac{m^2}{4b^2} - \frac{9}{16}$ . Since  $n < 1$  there is just one bound state in the mass spectrum of KK metric fluctuations: the zero mode massless state which accounts for the massless 4D graviton. To the best of our knowledge, this is the first model that presents just a *single* bound state in this kind of spectra within the framework of thick braneworlds. Usually one encounters a second bound state which represents a massive KK excitation with no clear physical interpretation (see [5], [24]–[29], for instance). The equation (38) can be integrated for arbitrary mass and possess the following general solution

$$\Psi(w) = C_1 P_{3/4}^\mu(\tanh(2bw)) + C_2 Q_{3/4}^\mu(\tanh(2bw)) \quad (40)$$

where  $C_1$  and  $C_2$  are constants, while  $P_{3/4}^\mu$  and  $Q_{3/4}^\mu$  are associated Legendre functions of first and second kind, respectively, with degree  $\nu = 3/4$  and order  $\mu = \sqrt{\frac{9}{16} - \frac{m^2}{4b^2}}$ . For the massless case the zero mode has  $\mu = \nu = 3/4$  and the exact solution can be expressed as

$$\Psi_0(w) = -k_1 \left[ P_{3/4}^{3/4}(\tanh(2bw)) + \frac{2}{\pi} Q_{3/4}^{3/4}(\tanh(2bw)) \right], \quad (41)$$

where now  $k_1 = -C_1 > 0$  and we have set  $C_2 = 2C_1/\pi$  in order to get a localized configuration. This bound state is physically interpreted as a stable graviton localized on the brane since there are no states with negative squared masses due to the structure of the potential (39). Finally, there is also a continuum of KK massive modes in the spectrum that starts from  $m \geq 3b/2$  and is described by eigenfunctions with imaginary order  $\pm\mu = \pm i\rho$  [26]:

$$\Psi_m(w) = \sum_{\pm} C_{\pm} P_{3/4}^{\pm i\rho}(\tanh(2bw)), \quad (42)$$

where  $C_{\pm}$  are arbitrary constants and  $\rho = \sqrt{\frac{m^2}{4b^2} - \frac{9}{16}}$ . These KK massive modes must behave as plane waves asymptotically [24, 25]. This fact can be seen by considering masses  $2m > 3b$  and taking into account that the constants  $C_{\pm}$  depend on  $\rho$  in general, i.e.

$$\Psi_{\pm}^\mu(w) = C_{\pm}(\rho) P_{3/4}^{\pm i\rho}(\tanh(2bw)). \quad (43)$$

We further make an expansion for large  $w$  of the argument of the associated Legendre functions of first kind:

$$\tanh(2bw) \simeq 1 - 2e^{-4bw} \quad (44)$$

and compute the asymptotic behaviour of  $P_{3/4}^{\pm i\rho}(\tanh(2bw))$  according to formula (8) of section 3.9.2 of [30]:

$$P_{3/4}^{\pm i\rho}(\tanh(2bw)) \sim \frac{1}{\Gamma(1 \mp i\rho)} e^{\pm 2ib\rho w}. \quad (45)$$

The normalization condition of (43) in the plane wave sense leads to the following normalization constants

$$C_+(\rho) = C_-(\rho) = \frac{|\Gamma(1+i\rho)|}{\sqrt{2\pi}} \quad (46)$$

since  $|\Gamma(1-i\rho)| = |\Gamma(1+i\rho)|$ . Thus, by substituting (45) and (46) into (43) we obtain the asymptotic behaviour of these associated Legendre functions:

$$\Psi_{\pm}^{\mu}(w) \sim \frac{1}{\sqrt{2\pi}} e^{\pm i2b\rho w} \quad (47)$$

which corresponds to plane waves as expected.

Once we have at hand an analytical expression for the KK massive modes, we should be able to compute the corresponding small corrections to Newton's law due to these 5D massive modes. This is achieved by taking the thin brane limit  $b \rightarrow \infty$ , locating a probe mass,  $M_1$ , in the center of the brane in the transverse direction and computing the gravitational potential generated by this particle that another massive particle with  $M_2$  realizes. The corrections to the Newtonian potential generated by massive gravitons in the thin brane limit can be expressed as follows [6]

$$U(r) \sim \frac{M_1 M_2}{r} \left( G_4 + M_*^{-3} \int_{m_0}^{\infty} dm e^{-mr} |\Psi^{\mu(m)}(w_0)|^2 \right) = \frac{M_1 M_2}{r} (G_4 + \Delta G_4), \quad (48)$$

where  $w = w_0$  sets the position where the brane is located,  $m_0 = 3b/2$  for our case,  $G_4$  is the gravitational 4D coupling constant and  $\Psi^{\mu}(w_0)$  denotes the continuum of KK massive modes that must be integrated over their masses in order to get the searched corrections.

Now we proceed to calculate  $|\Psi^{\mu}(0)|^2$  at  $w_0 = 0$  and get

$$|\Psi^{\mu}(0)|^2 = \left| \frac{\Gamma(1+i\rho)}{\Gamma\left(\frac{11}{8} + \frac{i\rho}{2}\right) \Gamma\left(\frac{1}{8} + \frac{i\rho}{2}\right)} \right|^2, \quad (49)$$

where we set  $\nu = 3/4$  and made use of the following relation for the associated Legendre functions of first kind [31]:

$$P_{\nu}^{\mu}(0) = \frac{2^{\mu} \sqrt{\pi}}{\Gamma\left(\frac{1-\nu-\mu}{2}\right) \Gamma\left(1 + \frac{\nu-\mu}{2}\right)}. \quad (50)$$

Substitution of (49) into the second term of (48) yields the following expression for  $\Delta G_4$  :

$$\Delta G_4 = M_*^{-3} \int_{m_0}^{\infty} dm e^{-mr} \left| \frac{\Gamma(1+i\rho)}{\Gamma\left(\frac{11}{8} + \frac{i\rho}{2}\right) \Gamma\left(\frac{1}{8} + \frac{i\rho}{2}\right)} \right|^2. \quad (51)$$

In order to calculate this integral it is convenient to perform a change of variable from  $m$  to  $\rho$ :

$$\Delta G_4 = 2M_*^{-3}b \int_0^\infty \frac{e^{-2\sqrt{\rho^2 + \frac{9}{16}}br}}{\sqrt{1 + \frac{9}{16\rho^2}}} \left| \frac{\Gamma(1 + i\rho)}{\Gamma(\frac{11}{8} + \frac{i\rho}{2})\Gamma(\frac{1}{8} + \frac{i\rho}{2})} \right|^2 d\rho. \quad (52)$$

This integral can be calculated in the thin brane limit  $b \rightarrow \infty$ , where it is dominated by the region of small  $\rho$ . Thus (52) can be well-approximated by expanding the factor that multiplies the exponential at  $\rho = 0$  [26]. Thus, the contribution to Newton's law made by the continuum of KK massive modes in the thin brane limit reads:

$$\Delta G_4 \sim \frac{M_*^{-3}}{|\Gamma(\frac{11}{8})\Gamma(\frac{1}{8})|^2} \frac{e^{-\frac{3}{2}br}}{r} \left( 1 + O\left(\frac{1}{br}\right) \right). \quad (53)$$

These corrections are exponentially suppressed as in other braneworld models with an induced 4D Minkowski [32], (see also [26]–[27]) or de Sitter metric [28, 29].

## 4 Discussion

In this paper we have presented a thick braneworld model generated by a tachyon scalar field coupled to gravity with a bulk cosmological constant and a de Sitter metric induced on the brane. We were able to obtain an exact solution with a decaying warp factor that gives rise to localization of 4D gravity when studying the metric fluctuations. The structure of the corresponding graviton spectrum is novel in the sense that it contains just one bound state (the massless zero mode) which is physically interpreted as the 4D graviton, separated by a mass gap from a continuum of KK massive excitations. We got an explicit expression for these KK massive modes, a fact that enables us to analytically compute the corrections to Newton's law coming from the extra dimension. As we mentioned above, these corrections are exponentially suppressed and coincide with previous results reported in the literature within the framework of similar braneworld models with the same 4D de Sitter induced metric.

Since we are considering a braneworld with an induced 4D de Sitter metric, in principle, we are able to reproduce the early inflation and accelerated expansion epochs of our universe within our model. However, a more general and realistic ansatz for our setup that attempts to describe the late time behaviour of our 4D universe should involve both a time-depending tachyon field and a time-depending warp factor, since from the cosmological viewpoint one needs to obtain scale factors that reproduce in a better way (closer to the observations) the accelerated expansion of the universe (which takes into account its dark matter component) than the de Sitter metric does. This line of research is under current investigation and could lead to interesting new results in cosmology.

## 5 Acknowledgements

GG and AHA thank SNI for support. AHA is grateful to U. Nucamendi for useful discussions and to the staff of the ICF, UNAM for hospitality; The research of AHA, DMM and RRML was supported by grant CONACYT 60060-J. DMM and RRML acknowledge a PhD grant from UMSNH and CONACYT, respectively. RR is grateful to E. Capelas de Oliveira for fruitful discussions and to Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) grants 476580/2010-2 and 304862/2009-6 for financial support.

## References

- [1] M. Gogberashvili, Hierarchy problem in the shell universe model, *Int. J. Mod. Phys. D* **11** (2002) 1635, hep-ph/9812296; Four dimensionality in noncompact Kaluza–Klein model, *Mod. Phys. Lett. A* **14** (1999) 2025, hep-ph/9904383.
- [2] L. Randall and R. Sundrum, A large mass hierarchy from a small extra dimension, *Phys. Rev. Lett.* **83** (1999) 3370, hep-ph/9905221; An alternative to compactification, *Phys. Rev. Lett.* **83** (1999) 4690, hep-th/9906064.
- [3] V. Dzhunushaliev, V. Folomeev and M. Minamitsuji, *Rept. Prog. Phys.* **73** (2010) 066901, arXiv:0904.1775 [gr-qc].
- [4] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, Modeling the fifth dimension with scalars and gravity, *Phys. Rev. D* **62** (2000) 046008, hep-th/9909134.
- [5] M. Gremm, Four–dimensional gravity on a thick domain wall, *Phys. Lett. B* **478** (2000) 434, hep-th/9912060; Thick domain walls and singular spaces, *Phys. Rev. D* **62** (2000) 044017, hep-th/0002040.
- [6] C. Csaki, J. Erlich, T. Hollowood and Y. Shirman, Universal Aspects of gravity localized on thick branes, *Nucl. Phys. B* **581** (2000) 309, hep-th/0001033.
- [7] M. Giovaninni, Gauge–invariant fluctuations of scalar branes, *Phys. Rev. D* **64** (2001) 064023, hep-th/0106041; Localization of metric fluctuations on scalar branes, *Phys. Rev. D* **65** (2002) 064008, hep-th/0106131; K. Farakos and P. Pasipoularides, Gauss–Bonnet gravity, brane world models, and non–minimal coupling, *Phys. Rev. D* **75** (2007) 024018, hep-th/0610010; K. Farakos, G. Koutsoumbas and P. Pasipoularides, Graviton localization and Newton’s law for brane models with a non-minimally coupled bulk scalar field, *Phys. Rev. D* **76** (2007) 064025, arXiv:0705.2364 [hep-th]; A. Herrera–Aguilar, D. Malagón–Morejón, R.R. Mora–Luna and I. Quiros, Thick braneworlds generated by a non-minimally coupled scalar field and a Gauss–Bonnet term: conditions for localization of gravity, *Class. Quantum Grav.* **29** (2012) 035012, arXiv:1105.5479 [hep-th]; H. Guo, Y.–X. Liu, Z.–H. Zhao and F.–W. Chen, Thick

- branes with a non-minimally coupled bulk-scalar field, Phys. Rev. D **85** (2012) 124033, arXiv:1106.5216 [hep-th].
- [8] S. Kobayashi, K. Koyama and J. Soda, Thick brane worlds and their stability, Phys. Rev. D **65** (2002) 064014, hep-th/0107025; V.I. Afonso, D. Bazeia, R. Menezes and A.Yu. Petrov, f(R)-Brane, Phys. Lett. B **658** (2007) 71, arXiv:0710.3790 [hep-th]; Y. Zhong, Y.-X. Liu and K. Yang, Tensor perturbations of f(R)-branes, Phys. Lett. B **699** (2011) 398, arXiv:1010.3478 [hep-th]; M. Gogberashvili and D. Singleton, Anti-de-Sitter Island-Universes from 5D Standing Waves, Mod. Phys. Lett. A **25** (2010) 2131, arXiv:0904.2828 [hep-th]; M. Gogberashvili, A. Herrera-Aguilar and D. Malagón-Morejón, An Anisotropic Standing Wave Braneworld and Associated Sturm-Liouville Problem, Class. Quantum Grav. **29** (2012) 025007, arXiv:1012.4534 [hep-th].
  - [9] O. Arias, R. Cardenas and I. Quiros, Thick Brane Worlds Arising From Pure Geometry, Nucl. Phys. B **643** (2002) 187, hep-th/0202130; N. Barbosa-Cendejas and A. Herrera-Aguilar, 4D gravity localized in non  $Z_2$ -symmetric thick branes, JHEP **0510** (2005) 101, hep-th/0511050; N. Barbosa-Cendejas and A. Herrera-Aguilar, Localization of 4D gravity on pure geometrical thick branes, Phys. Rev. D **73** (2006) 084022; *Erratum-ibid.* D **77** (2008) 049901, hep-th/0603184.
  - [10] J.M. Hoff da Silva and R. da Rocha, Braneworld remarks in Riemann-Cartan manifolds, Class. Quantum Grav. **26** (2009) 055007, *Erratum-ibid.* **26** (2009) 179801, arXiv:0804.4261 [gr-qc]; *ibid.* Torsion Effects in Braneworld Scenarios, Phys. Rev. D **81** (2010) 024021, arXiv:0912.5186 [hep-th]; *ibid.* Gravitational constraints of dS branes in AdS Einstein-Brans-Dicke bulk, Class. Quantum Grav. **27** (2010) 225008, arXiv:1006.5176 [gr-qc]; *ibid.* Effective Monopoles within Thick Branes, to appear in Europhys. Lett. (2012), arXiv:1209.0989 [hep-th].
  - [11] D. Bazeia, F.A. Brito and J.R. Nascimento, Supergravity brane worlds and tachyon potentials, Phys. Rev. D **68** (2003) 085007, hep-th/0306284.
  - [12] R. Koley and S. Kar, A Novel braneworld model with a bulk scalar field, Phys. Lett. B **623** (2005) 244, *Erratum-ibid.* B **631** (2005) 199, hep-th/0507277.
  - [13] D. Maity, S. SenGupta, and S. Sur, Stability analysis of the Randall-Sundrum braneworld in presence of bulk scalar, Phys. Lett. B **643** (2006) 348, hep-th/0604195.
  - [14] S. Pal and S. Kar, de Sitter branes with a bulk scalar, Gen. Rel. Grav. **41** (2009) 1165, hep-th/0701266.
  - [15] A. Das, S. Kar and S. SenGupta, Stable two-brane models with bulk tachyon matter, Int. J. Mod. Phys. A **24** (2009) 4457, arXiv:0804.1757 [hep-th].

- [16] W.D. Goldberger and M.B. Wise, Modulus stabilization with bulk fields, Phys. Rev. Lett. **83** (1999) 4922, hep-ph/9907447.
- [17] X.-H. Zhang, Y.-X. Liu, and Y.-S. Duan, Localization of Fermionic Fields on Braneworlds with Bulk Tachyon Matter, Mod. Phys. Lett. A**23** (2008) 2093, arXiv:0709.1888 [hep-th].
- [18] A. Sen, Rolling tachyon, JHEP **0204** (2002) 048, hep-th/0203211; *ibid.* Tachyon matter, JHEP **0207** (2002) 065, hep-th/0203265; *ibid.* Field theory of tachyon matter, Mod. Phys. Letts. A**17** (2002) 1797, hep-th/0204143; *ibid.* Time and tachyon, Int. J. Mod. Phys. A**18** (2003) 4869, hep-th/0209122.
- [19] P.D. Mannheim, *Brane-Localized Gravity*, World Scientific, Singapore (2005).
- [20] Ch. Charmousis, R. Gregory, N. Kaloper and A. Padilla, DGP spectroscopy, JHEP **0610** (2006) 066, hep-th/0604086.
- [21] G. 't Hooft, *Introduction to General relativity*, Rinton Press, Princeton (2001).
- [22] S. Carroll, *Spacetime and geometry: An introduction to general relativity*, Addison-Wesley, San Francisco (2004).
- [23] C. Gabriel and P. Spindel, Massive spin-2 propagators on de Sitter space, J. Math. Phys. **38** (1997) 622, hep-th/9912054; J. Garriga and M. Sasaki, Brane world creation and black holes, Phys. Rev. D**62** (2000) 043523, hep-th/9912118; T. Garidi, J.P. Gazeau and M.V. Takook, 'Massive' spin two field in de Sitter space, J. Math. Phys. **44** (2003) 3838, hep-th/0302022. J.P. Gazeau and M. Novello, The question of Mass in (anti-) de Sitter Spacetimes, J. Phys. A**41** (2008) 304008.
- [24] A. Wang, Thick de Sitter 3-Branes, Dynamic Black Holes and Localization of Gravity, Phys. Rev. D**66** (2002) 024024, hep-th/0201051.
- [25] M.K. Parikh and S.N. Solodukhin, De Sitter brane gravity: From close-up to panorama, Phys. Lett. B**503** (2001) 384, hep-th/0012231.
- [26] N. Barbosa-Cendejas, A. Herrera-Aguilar, M.A. Reyes Santos and C. Schubert, Mass gap for gravity localized on Weyl thick branes, Phys. Rev. D**77** (2008) 126013, arXiv:0709.3552 [hep-th].
- [27] N. Barbosa-Cendejas, A. Herrera-Aguilar, K. Kanakoglou, U. Nucamendi and I. Quiros, Mass hierarchy and mass gap on thick branes with Poincaré symmetry, arXiv:0712.3098[hep-th]; A. Herrera-Aguilar, D. Malagón-Morejón, R.R. Mora-Luna, U. Nucamendi, Aspects of thick brane worlds: 4D gravity localization, smoothness, and mass gap, Mod. Phys. Lett. A**25** (2010) 2089, arXiv:0910.0363 [hep-th].

- [28] A. Herrera–Aguilar, D. Malagón–Morejón and R.R. Mora–Luna, Localization of gravity on a thick braneworld without scalar fields, *JHEP* **1011** (2010) 015, arXiv:1009.1684[hep-th]; H. Guo, A. Herrera–Aguilar, Y.–X. Liu, D. Malagón–Morejón and R.R. Mora–Luna, Localization of bulk matter fields on a pure de Sitter thick braneworld, arXiv:1103.2430 [hep-th].
- [29] H. Guo, Y.–X. Liu, S.–W. Wei and C.–E. Fu, Gravity Localization and Effective Newtonian Potential for Bent Thick Branes, *Europhys. Lett.* **97** (2012) 60003, arXiv:1008.3686 [hep-th].
- [30] A. Erdélyi (Ed.), Higher transcendental functions, Vol. I, McGraw–Hill, New York (1953), reprint edition R.E. Krieger Publishing Company, Malabar (1981).
- [31] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products*, Academic Press, San Diego, (2007).
- [32] A. Brandhuber and K. Sfetsos, Nonstandard compactifications with mass gaps and Newton’s law, *JHEP* **10** (1999) 013, arXiv:hep-th/9908116.